The particle - particle interaction model

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- In order to relate the macroscopic equation-of-state observables such as radius and density of the 2D plasma crystal, we must have a relation connecting them to the particle interactions, which are assumed to be a Debye-screened Coulomb potential,

$$V_{pair}(r) = (q^2 / 4\pi\varepsilon_o)\exp(-r/\lambda)/r$$

- The Debye screening length is given by $1 / \lambda_d^2 = 1 / \lambda_e^2 + 1 / \lambda_i^2$ where $\lambda_{e,I}^2 = \varepsilon_0 \, k \, T_{e,I} / e^2 \, n_{e,I}$. For most conditions, the Debye screening length is dominated by the *ion* Debye length.

- The radial component of the gravitational force on a particle located at a radius *r* within the parabolic potential well is

$$f_r = -kr \qquad k = m_d \, g / R_c \qquad m_d = dust\ mass$$

- The number density for a 2D hcp lattice is

$$n(r) = 2/\sqrt{3}\, s(r)^2$$ where *s(r)* is the nearest neighbor (nn) separation at *r*.

- The pressure within a hcp lattice is

$$p = -\sqrt{3} V'_{pair}(s)/s$$ where *V'(r)* is the first spatial derivative.

- Using the Euler equation $dp/dr = n\,f_r = -k\,n\,r$ in the continuum-mechanics limit, the ordinary differential equation connecting the variation of nearest neighbor spacing to radius was derived:

$$(2/3)\,k\,r\,dr = (s\,V''_{pair}(s) - V'_{pair}(s))\,ds \qquad \text{Eq. 1}$$

- This equation is integrable to give an explicit *r(s, s₀)* where s_0 is the nearest neighbor separation at the center of the crystal.

- Additional useful relations are integrals derived from Equation 1 that connect the total number of particles N_{tot} and the radius of the outermost layer r_{max} to the nearest neighbor spacing in the center of the crystal s_0:

$$(k/2\pi)\,N_{tot} = -\sqrt{3}\; V'_{pair}(s_o)/s_o \qquad \text{Eq. 2}$$

$$(k/3)(r_{max} + s_{max}\sqrt{3}/2)^2 = 2V_{pair}(s_o) - s_o\,V'_{pair}(s_o) \qquad \text{Eq. 3}$$